Kinetic equations and turbulence

Large number multi-agent systems: mean field multiple/infinite-wise interactions and Hewitt-Savage theorem, and universal particle approximation of PDEs

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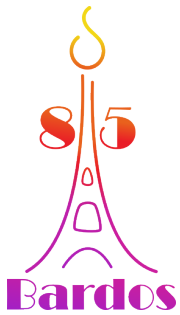
We consider multi-agent systems; i.e. systems of N non-indistinguishable particles, at microscopic, mesoscopic/kinetic and macroscopic/hydrodynamic scales. In the first part of the lecture we consider agents subject to multiple-wise interactions, i.e. when each particle interacts with m other ones at the same time. We derive the associated Vlasov equation and prove propagation of chaos. We then consider the hydrodynamic limit for monokinetic solution and derive the corresponding Euler equation. The precise estimates in N and m of the rate of convergence allows to consider the joint (conditional) limit of diverging N and m towards a new type of macroscopic equation involving a vector field derived out of the Hewit-Savage theorem and an unpublished result by Pierre-Louis Lions. In the second part, based on various ways to pass to the limit in multi-agent systems with distinguishable particles, using in particular the concept of graph limit, we show that sufficiently regular solutions of any well-posed quasilinear PDE can be approximated by solutions of systems of N particles, to within $1/\log(N)$.



In honor of Claude Bardos's 85th birthday