The extended magnetohydrodynamic equations

Nicolas Besse

Depending on the degree of precision retained at small scales, fluid models for plasma description are available in ideal MHD, Hall-MHD, up to extended MHD (sometimes called inertial and/or bi-fluid). Compared to the compressible ideal MHD, extended MHD contains supplementary physical effects such as the Hall effect and the inertial effect of electrons. Therefore, extended MHD provides a more accurate description of plasma dynamics (valid up to the scale of electron skin depth) and gives access to a wider range of applications, including a better understanding of inertial waves, fast magnetic reconnection (a process causing sawtooth oscillations in fusion devices as well as solar flares) and plasma turbulence. In the absence of resistivity, the Cauchy problem for the Hall-MHD is poorly posed (for example in the sense of Gevrey, see the result of I.-J. Jeong and of S.-J. Oh, published in Annals of PDE in 2022). The objective of the presentation is to explain how inertia effects restore stability. The goal is to present recent work (in collaboration with C. Cheverry) allowing a better understanding of the inertia regime through the study of systems of hyperbolic and/or parabolic type (penalized or not) resulting from this. More precisely, we first present the well-posedness theory for the extended MHD equations in the framework of classical solutions. Second, we expose the asymptotic analysis of the extended MHD equations in the case where the plasma is strongly magnetized and anisotropic. In this asymptotic framework, we also present the limit of weak solutions of the standard compressible MHD to those of the reduced MHD, which turns out to be a mixed incompressible-compressible model. Such efforts to justify rigorously these reduced MHD models are motivated by their extensive use in fusion, space and astrophysical plasmas.

In honor of Claude Bardos's 85th birthday