Poster session

Tuesday, April 8th 2025 from 4:10 PM

List of participants:

- Michael Abdelmalik (Eindhoven University of Technology)
- Daniel Boutros (University of Cambridge)
- Emile Breton (Université de Rennes)
- Marie Farge (ENS Ulm)
- Jakob Möller (École Polytechnique)
- Lukas Niebel (University of Münster)
- Chenjiayue Qi (IHÉS)
- Matthias Rakotomalala (École Polytechnique)
- Marwa Shahine (Eindhoven University of Technology)

Extensions to the Navier-Stokes-Fourier Equations for Rarefied Transport

Michael Abdelmalik[1]

Joint work with Frimpong Baidoo[2]*, Thomas Hugues*[2] *and Irene Gamba*[2]

We present an entropy-stable extension of the Navier-Stokes-Fourier equations into the transition regime of rarefied gases. Our approach is based on a novel variational reformulation of the closure problem, derived from the Boltzmann equation, which subsumes existing methods such as Chapman-Enskog. We demonstrate the effectiveness of our closure model in capturing rarefied transport phenomena by comparing analytical solutions of our closed conservation equations to solutions of the Boltzmann equation for benchmark problems.

[1]Eindhoven University of Technology, Netherlands.
[2]Oden Institute, University of Texas at Austin, United States of America.

Hölder regularity of the pressure for weak solutions of the 3D Euler equations in bounded domains

Daniel Boutros[1]

Joint work with Claude Bardos[2] *and Edriss S. Titi*[1]

Onsager's conjecture provides regularity conditions for weak solutions of the incompressible Euler equations to conserve energy, which is related to the development of turbulence in the zero viscosity limit. This conjecture has been fully established in the case of periodic boundary conditions. We consider Onsager's conjecture in the case of a bounded domain, as boundary effects play a crucial role in hydrodynamic turbulence. As part of the treatment of this problem, we rigorously derive the boundary-value problem for the hydrodynamic pressure for weak solutions of the Euler equations and prove a regularity result in Hölder spaces, which requires the introduction and rigorous derivation of a new boundary condition. We prove this new boundary condition to be necessary by means of an explicit example.

[1]University of Cambridge, England.
[2]Université Paris Cité, France.

In honor of Claude Bardos's 85th birthday

The Vlasov-Maxwell system: non L^1 asymptotic completeness and modified scattering for compactly supported data

Emile Breton[1]

We consider the solutions to the Vlasov-Maxwell system exhibited by Glassey and Strauss, arising from small and compactly supported data. We prove that the particle densities exhibit a modified scattering dynamic in the multi-species case. Moreover, in this setting, we show the non L^1 asymptotic completeness i.e. there exists at least one particle density that does not converge in L^1.

[1]Université de Rennes, France.

In honor of Claude Bardos's 85th birthday

Production of dissipative vortices by solid bodies in incompressible fluid flows: comparison between Prandtl, Navier-Stokes and Euler solutions

Marie Farge[1]

Joint work with Romain Nguyen van yen[2]*, Matthias Waidmann*[2]*, Kai Schneider*[3] *and Rupert Klein*[2]

We propose to revisit the problem posed by Euler in 1748 for the Prize of Mathematics set by the Prussian Academy of Sciences concerning the resistance that fluid flows exert on solid bodies, which led to the 'd'Alembert's paradox'. In 1904 Prandtl introduced the notion of boundary layer, assuming all viscous energy dissipation takes place only in the boundary layer, as long as it remains in contact with the body, and proposed a methodology to resolve d'Alembert's paradox in this case. Our poster will address the following problem: does energy dissipate when the boundary layer detaches from the solid body and how this happens?

[1]École Normale Supérieure de Paris, France.
[2]Freie Universität Berlin, Germany.
[3]Université d'Aix-Marseille, France.

(Semi-classical) Analysis of Self-consistent Semi-relativistic Pauli equations

Jakob Möller[1]

Joint work with Norbert J. Mauser[2]

We present self-consistent models in (semi-)relativistic quantum mechanics, i.e. nonlinear systems of PDE describing a particle coupled to PDE describing the electromagnetic field created by a moving charge which self-interacts with the particle. In the hierarchy ranging from the relativistic Dirac-Maxwell equation to the non-relativistic Vlasov-Poisson equation, the semi-relativistic Pauli equation for a 2-spinor, coupled to first and second order approximations of the Maxwell equations (i.e. the Poisswell and Darwin equations) is the first consistent model that includes both relativistic effects of magnetic field and spin. We present the full hierarchy including "semi-classical" limits (where the Planck constant tends to zero) towards Vlasov and Euler equations and the first rigorous results on the (asymptotic) analysis of self-consistent Pauli equations as well as various related nonlinear PDE.

[1]École Polytechnique, France.
[2]Wolfgang Pauli Institut c/o Department of Mathematics, Universität Wien, Austria.

Kinetic trajectories

Lukas Niebel[1]

Kinetic trajectories link the geometry of kinetic equations to strong regularity properties. This poster outlines their construction and applications to regularity theory.

[1]University of Münster, Germany.

Kinetic equations and turbulence

Global Solution of a Functional Hamilton-Jacobi Equation associated with a Hard Sphere Gas

Chenjiayue Qi[1]

In recent years it has been shown for hard sphere gas that, by retaining the correlation information, dynamical fluctuation and large deviation of empirical measure around Boltzmann equation can be proved, in addition to the classical kinetic limit result by Lanford. After taking low-density limit, the correlation information can be encoded into a functional Hamilton-Jacobi equation. The results above are restricted to short time. This paper deals with global-in-time behavior of a solution of the Hamilton-Jacobi equation, by analyzing a system of coupled Boltzmann equations.

[1]IHÉS, France.

In honor of Claude Bardos's 85th birthday

Analysis of a PDE Model for Ant Trail Formation

Matthias Rakotomalala[1]

Joint work with Charles Bertucci[1]*, Milica Tomašević*[1]
and Oscar de Wit[2]

We propose a new model of chemotaxis motivated by ant trail pattern formation, formulated as a coupled parabolic-parabolic PDE system, for the population density and the chemical field. The main novelty lies in the transport term of the population density, which depends on the second-order derivatives of the chemical field. This term is derived as an anticipation-reaction steering mechanism of an infinitesimally small ant as its size approaches zero. We establish global-in-time existence and uniqueness for the model, as well as the propagation of regularity from the initial data. We then establish the existence of a compact global attractor and prove the nonlinear instability of the homogeneous steady state under an inviscid instability condition. We also provide a dimensional lower bound on the attractor.

[1]École Polytechnique, France.
[2]University of Cambridge, England.

Polyatomic Boltzmann Equation: Entropy-Based Ansatz for Galerkin Approximations

Marwa Shahine[1]

Joint work with Michael Abdelmalik[1] *and Gieling Bas*[1]

In this work, we consider the Boltzmann equation modeling polyatomic gases, proposed by Bourgat et al. (1994). We propose an approximation of the distribution function based on Phi-divergences, and we prove that this approximation yields entropy stable closures leading to symmetric hyperbolic momentsystems. We then compare the properties of the proposed approximation to the Discontinuous Galerkin method to available numerical results.

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