Polarized Light in a Stratified Atmosphere with a Varying Refractive Index

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This talk is an attempt to elucidate the effect of a varying refractive index on the temperature in a stratified atmosphere, with a particular focus on greenhouse gases such as $\mathtt{CO}_2$. It validates an iterative method for the vector refractive radiative transfer equations (VRRTE) called Iterations on the Source. A proof is given showing monotonicity, convergence of the iterations and existence and uniqueness when all rays cross the atmosphere. The analysis is extended to Fresnel's law on one flat discontinuity of the refraction index. Numerical simulations will be show for an ocean-atmosphere configuration.

In honor of Claude Bardos's 85th birthday

Light and Polarization Intensities are I and Q. The refractive index is $n(z)$ and the tilde indicates a division by n^2. $B_\nu(T) = \nu^3/(e^{\frac{\nu}{T}} - 1)$ is the (rescaled) Plank function of temperature T and frequency ν. We analyze and solve numerically the following

$$\begin{cases} \mu\partial_z \tilde{I} + (\partial_z \log n)(1-\mu^2)\partial_\mu \tilde{I} + \kappa\tilde{I} \\ \qquad = \kappa_a \tilde{B}_\nu(T) + \dfrac{\kappa_s}{2}\displaystyle\int_{-1}^{1} \tilde{I} d\mu' + \dfrac{\beta\kappa_s}{4} P_2(\mu) \displaystyle\int_{-1}^{1} [P_2\tilde{I} - (1-P_2)\tilde{Q}] d\mu', \\ \mu\partial_z \tilde{Q} + \partial_z \log n(1-\mu^2)\partial_\mu \tilde{Q} + \kappa\tilde{Q} \\ \qquad = -\dfrac{\beta\kappa_s}{4}(1 - P_2(\mu)) \displaystyle\int_{-1}^{1} [P_2\tilde{I} - (1-P_2)\tilde{Q}] d\mu', \end{cases}$$

where $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$, κ is absorption, κ_s is scattering and $\kappa_a = \kappa - \kappa_s$. The domain is $\Omega = (-1, 1) \times (0, Z)$.

We consider the following boundary conditions:

$$I(\mu, 0)|_{\mu>0} = I_0, \quad I(\mu, Z)|_{\mu<0} = 0.$$

The temperature is given in terms of the wind velocity $\mathbf{u}$ and thermal diffusion κ_T by

$$\mathbf{u} \cdot \nabla T - \kappa_T \Delta T + \int_0^\infty \kappa_a \big[\tilde{B}_\nu(T) - \tfrac{1}{2}\int_{-1}^{1} \tilde{I} \mathrm{d}\mu\big] \mathrm{d}\nu = 0.$$